

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751**: Introduction to Advanced Mathematics

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is a guide to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but will also be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect working. This work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme but is according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Full marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient to just indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors and the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be awarded if an associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working for a correct answer is not ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply if a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate a mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate goes once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and should be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously awarded marks. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (or B) mark is not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable is given in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks are given 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if it is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures as a general norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is the norm) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in no mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be given in the mark scheme rationale. If in doubt, contact your Team Leader.

- g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then the mark should be given for that attempt. Do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question are the same, the candidate is awarded the full mark according to the scheme but following through from the candidate's data. A penalty is then applied to the mark if appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1		$y = -0.5x + 3$ oe www isw	3 [3]	B2 for $2y = -x + 6$ oe or M1 for gradient = $-\frac{1}{2}$ oe seen or used and M1 for $y - 1 = \text{their } m(x - 4)$ for or M1 for $y =$ substituted
2		substitution to eliminate one variable simplification to $ax = b$ or $ax - b = 0$ form, or equivalent for y (0.7, 0.1) oe or $x = 0.7, y = 0.1$ oe isw	M1 M1 A2 [4]	or multiplication to make one pair of coefficients the same; condone one error in either method or appropriate subtraction / addition; condone one error in either method A1 each independent of
3	(i)	25	2 [2]	M1 for $\left(\frac{10}{2}\right)^2$ or $\left(\frac{1}{0.2}\right)^2$ oe soi or for $\frac{1}{0.04}$ oe ie M1 for one correctly M0 for just $\frac{1}{0}$
3	(ii)	$8a^9$	3 [3]	B2 for 8 or M1 for $16^{\frac{1}{2}} = 2$ soi and B1 for a^9 ignore \pm eg M1 for 2^3 ;

4		$r = \sqrt{\frac{3V}{\pi(a+b)}}$ oe www as final answer	3 [3]	M1 for dealing correctly with 3 and M1 for dealing correctly with $\pi(a+b)$, ft and M1 for correctly finding square root, ft <i>their</i> ' $r^2 =$ '; square root symbol must extend below the fraction line	le-d w. condon M0 if $\pm \dots$ or for M3, final
5		$f(2) = 18$ seen or used $32 + 2k - 20 = 18$ oe $[k =] 3$	M1 A1 A1 [3]	or long division oe as far as obtaining a remainder (ie not involving x) and equating that remainder to 18 (there may be errors along the way) after long division: $2(k+16) - 20 = 18$ oe	A0 for just 2^5 implied by fu

6		-2560 www	4	<p>B3 for 2560 from correct term (NB coefficient of x^4 is 2560)</p> <p>or B3 for neg answer following $10 \times 4 \times -64$ and then an error in multiplication</p> <p>or M2 for $10 \times 2^2 \times (-4)^3$ oe; must have multn signs or be followed by a clear attempt at multn;</p> <p>or M1 for $2^2 \times (-4)^3$ oe (condone missing brackets) or for 10 used or for 1 5 10 10 5 1 seen</p> <p>for those who find the coefft of x^2 instead: allow M1 for 10 used or for 1 5 10 10 5 1 seen ; and a further SC1 if they get 1280, similarly for finding coefficient of x^4 as 2560</p>	<p>ms 1</p> <p>x</p> <p>but eg 1</p> <p>gets M2 on</p> <p>condone miss</p> <p>for $10 \times 2^2 \times$</p> <p>5C_3 or factoria</p> <p>sufficient but</p> <p>10 may be un</p> <p>M1 only for e</p> <p>table with no</p> <p>of attempt at</p> <p>[lack of neg s</p> <p>means that th</p> <p>eligible for ju</p>
7	(i)	$5^{3.5}$ oe or $k = 7/2$ oe	<p>2</p> <p>[2]</p>	M1 for $125 = 5^3$ or $\sqrt{5} = 5^{\frac{1}{2}}$ soi	M0 for just a reference to 1

7	(ii)	<p>attempting to multiply numerator and denominator of fraction by $1 + 2\sqrt{5}$</p> <p>denominator = -19 soi</p> <p>$8 + 3\sqrt{5}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>must be obtained correctly, but independent of first M1</p>	<p>ds a</p> <p>10</p> <p>are av</p> <p>wrongly c</p> <p>multiplied by</p> <p>eg M1 for den</p> <p>minus sign in</p> <p>or with attem</p> <p>numerator</p>
8		<p>$3(x - 2)^2 - 7$ isw or $a = 3, b = 2, c = 7$ www</p> <p>-7 or ft</p>	<p>4</p> <p>B1</p> <p>[5]</p>	<p>B1 each for $a = 3, b = 2$ oe</p> <p>and B2 for $c = 7$ oe</p> <p>or M1 for $[-\frac{7}{3}]$ or for $5 - \text{their } a(\text{their } b)^2$</p> <p>or for $\frac{5}{3} - (\text{their } b)^2$ soi</p> <p>B1</p> <p>B0 for $(2, -7)$</p>	<p>condone omis</p> <p>ignore '= 0'</p> <p>may be implic</p> <p>may be obtain</p> <p>with calculus</p>
9	(i)	<p>$3n$ isw</p>	<p>1</p> <p>[1]</p>	<p>accept equivalent general explanation</p>	

9	(ii)	<p>at least one of $(n - 1)^2$ and $(n + 1)^2$ correctly expanded</p> <p>$3n^2 + 2$</p> <p>comment eg $3n^2$ is always a multiple of 3 so remainder after dividing by 3 is always 2</p>	<p>M1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>must be seen</p> <p>dep on previous B1</p> <p>B0 for just saying that 2 is not divisible by 3 – must comment on $3n^2$ term as well</p> <p>allow B1 for $\frac{3n^2 + 2}{3} = n^2 + \frac{2}{3}$</p>	<p>accept e expansions</p> <p>SC: $n, n + 1, n + 2$ obtain first M similar comm</p>
10	(i)	<p>[radius =] $\sqrt{20}$ or $2\sqrt{5}$ isw</p> <p>[centre =] (3, 2)</p>	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>B0 for $\pm\sqrt{20}$ oe</p>	<p>condone lack coordinates, h</p>

10	(ii)	<p>substitution of $x = 0$ or $y = 0$ into circle equation</p> <p>$(x - 7)(x + 1) [=0]$</p> <p>$(7, 0)$ and $(-1, 0)$ isw</p> <p>$[y =] \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times (-7)}}{2}$ oe</p> <p>$(0, 2 \pm \sqrt{11})$ or $\left(0, \frac{4 \pm \sqrt{44}}{2}\right)$ isw</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>or use of Pythagoras with radius and a coordinate of the centre eg $20 - 2^2$ or $h^2 + 3^2 = 20$ ft their centre and/or radius</p> <p>no ft from wrong quadratic; for factors giving two terms correct, or formula or completing square used with at most one error</p> <p>accept $x = 7$ or -1 (both required)</p> <p>no ft from wrong quadratic; for formula or completing square used with at most one error</p> <p>accept $y = \frac{4 \pm \sqrt{44}}{2}$ oe isw</p>	<p>may a</p> <p>m</p> <p>bod into</p> <p>allow M1 for $(y - 2)^2 = 20$</p> <p>completing square at least $(x - a)$</p> <p>following use for attempt to</p> <p>completing square at least $(y - a)$</p> <p>following use for attempt to</p> <p>annotation is earned in this each mark ear</p>
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10	(iii)	<p>show both A and B are on circle</p> <p>(4, 5)</p> <p>$\sqrt{10}$</p>	<p>B1</p> <p>B2</p> <p>B2</p> <p>[5]</p>	<p>explicit substitution in circle equation and at least one stage of interim working required oe</p> <p>B1 each or M1 for $\left(\frac{7+1}{2}, \frac{6+4}{2}\right)$</p> <p>from correct midpoint and centre used; B1 for $\pm\sqrt{10}$</p> <p>M1 for $(4-3)^2 + (5-2)^2$ or $1^2 + 3^2$ or ft their centre and/or midpoint, or for the square root of this</p>	<p>use of and</p> <p>may be a long of $\frac{1}{2}$ AB and</p> <p>no ft if one co as that of cent formula/Pytha centre correct</p> <p>annotation is earned in this each mark ear</p>
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11	(i)	<p>sketch of cubic the right way up, with two tps and clearly crossing the x axis in 3 places</p> <p>crossing/reaching the x-axis at $-4, -2$ and 1.5</p> <p>intersection of y-axis at -24</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>intersections must be shown correctly labelled or worked out nearby; mark intent</p>	<p>on to</p> <p>ba</p> <p>at en</p> <p>turning</p> <p>doubling</p> <p>continue to s</p> <p>y-axis or in 3</p> <p>must clearly</p> <p>axis at both</p> <p>accept curve</p> <p>between 1 and</p> <p>NB to find -2</p> <p>$f(x)$ here, whi</p> <p>this is done, p</p> <p>by (iii)A to al</p> <p>appears again</p>
11	(ii)	<p>$-2, 0$ and $7/2$ oe isw or ft their intersections</p>	<p>2</p> <p>[2]</p>	<p>B1 for 2 correct or ft or for $(-2, 0)$ $(0, 0)$ and $(3.5, 0)$ or M1 for $(x + 2) x (2x - 7)$ oe or SC1 for $-6, -4$ and $-1/2$ oe</p>	

11	(iii)	(B)	<p>$g(1) = 2 + 9 - 2 - 9 [=0]$</p> <p>attempt at division by $(x - 1)$ as far as $2x^3 - 2x^2$ in working</p> <p>correctly obtaining $2x^2 + 11x + 9$</p> <p>factorising a correct quadratic factor</p> <p>$(2x + 9)(x + 1)(x - 1)$ isw</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>allow this mark for $(x - 1)$ shown to be a factor and a statement that this means that $x = 1$ is a root [of $g(x) = 0$] oe</p> <p>or inspection with at least two terms of quadratic factor correct</p> <p>allow B2 for another linear factor found by the factor theorem</p> <p>for factors giving two terms correct; eg allow M1 for factorising $2x^2 + 7x - 9$ after division by $x + 1$</p> <p>allow $2(x + 9/2)(x + 1)(x - 1)$ oe; dependent on 2nd M1 only; condone omission of first factor found; ignore '= 0' seen</p>	<p>st g(</p> <p>9 l</p> <p>M0 for div. unless further 0 shown, but M1A1</p> <p>NB mixture of this part – ma eg three uses or two uses p factor;</p> <p>allow M1 for -1 and -18/4 formula</p> <p>SC alternative allow first M and then seco factorisation</p>
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12	(i)	$y = 2x + 3$ drawn accurately (-1.6 to -1.7, -0.2 to -0.3) (2.1 to 2.2, 7.2 to 7.4)	M1 B1 B1 [3]	at least as far as intersecting curve twice intersections may be in form $x = \dots, y = \dots$	if marking by relevant to (ii) and in (ii) to a
12	(ii)	$\frac{1}{x-2} = 2x+3$ $1 = (2x+3)(x-2)$ $1 = 2x^2 - x - 6$ oe $\frac{1 \pm \sqrt{1^2 - 4 \times 2 \times -7}}{2 \times 2}$ oe $\frac{1 \pm \sqrt{57}}{4}$ isw	M1 M1 A1 M1 A1 [5]	or attempt at elimination of x by rearrangement and substitution condone lack of brackets for correct expansion; need not be simplified; NB A0 for $2x^2 - x - 7 = 0$ without expansion seen [given answer] use of formula or completing square on given equation, with at most one error isw eg coordinates; after completing square, accept $\frac{1}{4} \pm \sqrt{\frac{57}{16}}$ or better	may be seen in part (i) work & image for (ii) rather than in implies first M implies second after $\frac{1}{x-2} = 2x+3$ completing square at least [2](x - stage oe with

12	(iii)	$\frac{1}{x-2} = -x + k$ and attempt at rearrangement $x^2 - (k+2)x + 2k + 1 [=0]$ $b^2 - 4ac = 0$ oe seen or used $[k =] 0$ or 4 as final answer, both required	M1 M1 M1 A1 [4]	for simplifying and rearranging to zero; condone one error; collection of x terms with bracket not required SC1 for 0 and 4 found if 3 rd M1 not earned (may or may not have earned first two Ms)	eg M1 be or M1 for x $= 0$ may not be implied by the eg obtained g calculus and/ range
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Appendix: revised tolerances for modified papers for visually impaired candidates (graph in 12(i) with ...)

12	(i)	$y = 2x + 3$ drawn accurately (-1.6 to -1.8, -0.2 to -0.3) (2.1 to 2.3, 7.1 to 7.4)	M1	at least as far as intersecting curve twice	ru... (2, 1)
			B1	intersections may be in form $x = \dots, y = \dots$	
			B1		
			[3]		if marking by relevant to (ii) and in (ii) to a

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